

确定会给的公式表

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}, \quad (1)$$

$$J_\nu(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(k+\nu+1)} \left(\frac{x}{2}\right)^{2k+\nu}, \quad (2)$$

$$N_\nu(x) = \lim_{\alpha \rightarrow \nu} \frac{J_\alpha(x) \cos(\alpha\pi) - J_{-\alpha}(x)}{\sin(\alpha\pi)}, \quad (3)$$

$$J_m(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(x \sin \theta - m\theta)} d\theta, \quad (m \in Z), \quad (4)$$

$$e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} J_n(x)t^n, \quad (5)$$

$$\int_0^1 x J_m(\mu_i x) J_m(\mu_j x) dx = \delta_{ij} \frac{[J_{m+1}(\mu_i)]^2}{2}, \quad (\text{这里 } \mu_1, \mu_2, \dots \text{ 是 } J_m \text{ 的正实数零点}), \quad (6)$$

$$\int_0^1 x J_m(\lambda_i x) J_m(\lambda_j x) dx = \delta_{ij} \frac{\left(1 - \frac{m^2}{\lambda_i^2}\right) [J_m(\lambda_i)]^2}{2}, \quad (\text{这里 } \lambda_1, \lambda_2, \dots \text{ 是 } J'_m \text{ 的正实数零点}), \quad (7)$$

$$\int_0^{\infty} J_m(k_1 r) J_m(k_2 r) r dr = \frac{\delta(k_1 - k_2)}{k_1}, \quad (k_1, k_2 > 0), \quad (8)$$

$$Y_{\ell m}(\theta, \phi) = \frac{1}{2^\ell \ell!} \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} \left[\sin^m \theta \left(\frac{1}{\sin \theta} \frac{d}{d\theta} \right)^{\ell+m} \sin^{2\ell} \theta \right] e^{im\phi}, \quad (9)$$

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}} J_{\ell+1/2}(x), \quad (10)$$

$$n_\ell(x) = \sqrt{\frac{\pi}{2x}} N_{\ell+1/2}(x), \quad (11)$$

$$j_l(x) = (-x)^l \left(\frac{1}{x} \frac{d}{dx} \right)^l \frac{\sin x}{x}, \quad (12)$$

$$n_l(x) = -(-x)^l \left(\frac{1}{x} \frac{d}{dx} \right)^l \frac{\cos x}{x}, \quad (13)$$

$$P_\ell(x) = \sum_{k=0}^{\ell} \frac{(\ell+k)!}{(k!)^2(\ell-k)!} \left(\frac{x-1}{2}\right)^k, \quad (14)$$

$$P_\ell(\cos \theta) = \sqrt{\frac{4\pi}{2\ell+1}} Y_{\ell 0}(\theta, \phi), \quad (15)$$

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell} (x^2-1)^\ell, \quad (16)$$

$$(2\ell+1)xP_\ell(x) = (\ell+1)P_{\ell+1}(x) + \ell P_{\ell-1}(x), \quad (17)$$

$$\frac{1}{\sqrt{1+t^2-2xt}} = \sum_{\ell=0}^{\infty} P_\ell(x)t^\ell, \quad (t < |x \pm \sqrt{x^2-1}|), \quad (18)$$

$$P_\ell(\mathbf{n}_1 \cdot \mathbf{n}_2) = \frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{\ell} Y_{\ell m}^*(\mathbf{n}_1) Y_{\ell m}(\mathbf{n}_2), \quad (19)$$